Subject :
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$$f_n = \left[1732^2 + 866^2 + 866^2\right]^{\frac{1}{2}} \qquad \tau_n = \left[f_n^2 - \sigma_{nn}^2\right]^{\frac{1}{2}}$$

$$\tau_n = 707 \ Kg/cm^2$$

در یک صفحه دلخواه (n) : (یعنی محورهای مختصات منطبق بر جهت‌های اصلی هستند.)

مطابق با صفحه ۷ جزوه

$$f_{nx} = \sigma_{xx} l_{nx} + \cancel{\tau_{yx} l_{ny}} + \cancel{\tau_{zx} l_{nz}}$$

$$f_n^2 = \tau_{nt}^2 + \sigma_{nn}^2 = l_{nx}^2 \sigma_1^2 + l_{ny}^2 \sigma_2^2 + l_{nz}^2 \sigma_3^2$$

$$\sigma_{nn} = l_{nx}^2 \sigma_1 + l_{ny}^2 \sigma_2 + l_{nz}^2 \sigma_3$$

$$l_{nx}^2 + l_{ny}^2 + l_{nz}^2 = 1$$

از ترکیب این روابط داریم :

$$l_{nx}^2 = \frac{\tau_{nt}^2 + (\sigma_{nn} - \sigma_2)(\sigma_{nn} - \sigma_3)}{(\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3)}$$

$$l_{ny}^2 = \frac{\tau_{nt}^2 + (\sigma_{nn} - \sigma_3)(\sigma_{nn} - \sigma_1)}{(\sigma_2 - \sigma_3)(\sigma_2 - \sigma_1)}$$

$$l_{nz}^2 = \frac{\tau_{nt}^2 + (\sigma_{nn} - \sigma_1)(\sigma_{nn} - \sigma_2)}{(\sigma_3 - \sigma_1)(\sigma_3 - \sigma_2)}$$

با فرض اینکه : $\sigma_1 > \sigma_2 > \sigma_3$

$$\begin{cases} \tau_{nt}^2 + (\sigma_{nn} - \sigma_2)(\sigma_{nn} - \sigma_3) \geq 0 \\ \tau_{nt}^2 + (\sigma_{nn} - \sigma_3)(\sigma_{nn} - \sigma_1) \leq 0 \\ \tau_{nt}^2 + (\sigma_{nn} - \sigma_1)(\sigma_{nn} - \sigma_2) \geq 0 \end{cases}$$

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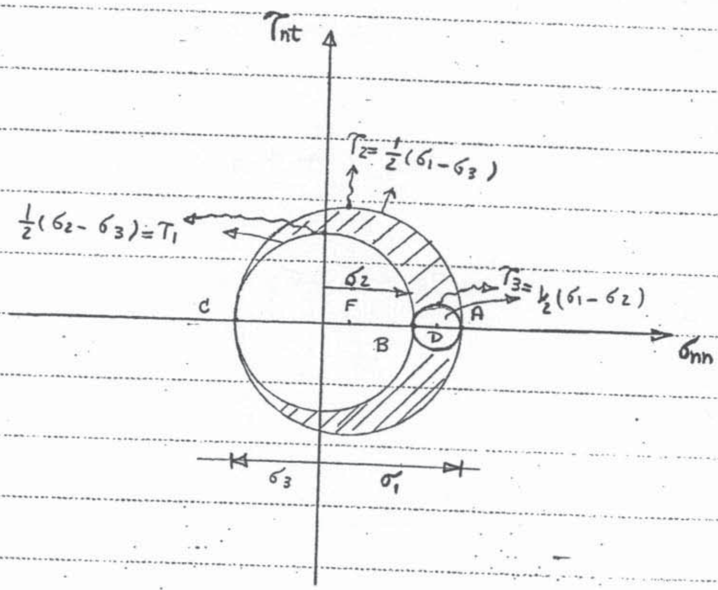
$$\tau_{nt}^2 + \left[\sigma_{nn} - \frac{1}{2}(\sigma_2 + \sigma_3)\right]^2 \geq \left[\frac{1}{2}(\sigma_2 - \sigma_3)\right]^2$$

$$\tau_{nt}^2 + \left[\sigma_{nn} - \frac{1}{2}(\sigma_3 + \sigma_1)\right]^2 \leq \left[\frac{1}{2}(\sigma_3 - \sigma_1)\right]^2$$

$$\tau_{nt}^2 + \left[\sigma_{nn} - \frac{1}{2}(\sigma_1 + \sigma_2)\right]^2 \geq \left[\frac{1}{2}(\sigma_1 - \sigma_2)\right]^2$$

قسمت هاشور خورده ؛ مقادیر قابل قبول



$$\tau_{nt}^2 + (\sigma_{nn} - \sigma_2)(\sigma_{nn} - \sigma_3) = l_{nx}^2 (\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3)$$

$$\tau_{nt}^2 + (\sigma_{nn} - \sigma_3)(\sigma_{nn} - \sigma_1) = l_{ny}^2 (\sigma_2 - \sigma_3)(\sigma_2 - \sigma_1)$$

$$\tau_{nt}^2 + (\sigma_{nn} - \sigma_1)(\sigma_{nn} - \sigma_2) = l_{nz}^2 (\sigma_3 - \sigma_1)(\sigma_3 - \sigma_2)$$

$$\hookrightarrow \tau_{nt}^2 + \left[\sigma_{nn} - \frac{1}{2}(\sigma_2 + \sigma_3)\right]^2 = l_{nx}^2 (\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3) + \left[\frac{1}{2}(\sigma_2 - \sigma_3)\right]^2 = R_1^2 \text{ (شعاع دایره)}$$

l_{nx}, l_{ny}, l_{nz}

، دایره مربوط به σ_2 و σ_3 ،

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$T = T_m + \underset{\downarrow}{T_d}$ تانسور تنش

تانسور deviator (منحرف ، انحرافی)

T_m تانسور اسفریکال (تانسور تنش میانگین)

$$\sigma_m = \frac{\sigma_{11}+\sigma_{22}+\sigma_{33}}{3} \qquad \sigma_m = \frac{\sigma_{xx}+\sigma_{yy}+\sigma_{zz}}{3}$$

* تانسور تنش deviator در شکست المانها موثر است. (T_m تاثیری ندارد)

یک مثال :

$$T = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix} + \begin{bmatrix} \frac{2\sigma_1-\sigma_2-\sigma_3}{3} & 0 & 0 \\ 0 & \frac{2\sigma_2-\sigma_1-\sigma_3}{3} & 0 \\ 0 & 0 & \frac{2\sigma_3-\sigma_1-\sigma_2}{3} \end{bmatrix}$$

T_m (باعث تغییر حجم میشود) همواره تنها دایره های قطر صفر هستند.

T_d (باعث تغییر شکل بدون تغییر حجم میشود)

* تنش اصلی همان ان واریان است.
(I)

$$I_{1m} = I_1 = 3\sigma_m \qquad I_{1d} = 0$$

$$I_{2m} = \frac{1}{3} I_1^2 = 3\sigma_m^2 \qquad I_{2d} = I_2 - \frac{1}{3} I_1^2 = \frac{-1}{6}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

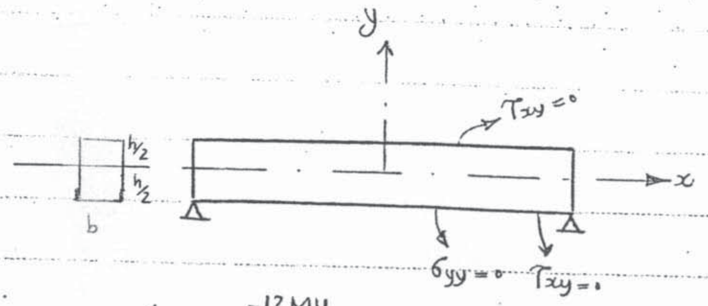
$$I_{3m} = \frac{1}{27} I_1^3 = \sigma_m^3 \qquad I_{3d} = \frac{1}{27}(2\sigma_1-\sigma_2-\sigma_3)(2\sigma_2-\sigma_3-\sigma_1)(2\sigma_3-\sigma_2-\sigma_1)$$

$$I_{1d} = 0 \qquad I_{3d} = I_3 - \frac{1}{3} I_1 I_2 + \frac{2}{27} I_1^3$$

برای ۱۸/۱ تمرین : ان واریان های اول و دوم تنش را بازنویسی کنید.

تمرین : تانسور تنش دویاتور و اسفریکال را از طریق تانسورهای تنش معمولی (حالت کلی) بنویسید و ان واریان های آنها را حساب کنید.

برای ۱۸ر تمرین، τ_{xy} و σ_{yy} را بدست آورید:



$$\sigma_{xx} = \frac{-12My}{bh^3}$$

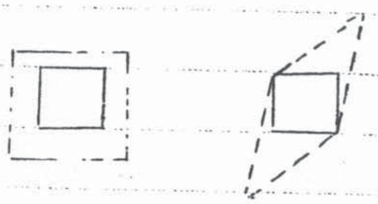
$M = M(x)$

$$\begin{cases} \sigma_{zz} = \tau_{xz} = \tau_{zy} = 0 \\ \text{نیروهای حجمی} = 0 \end{cases}$$

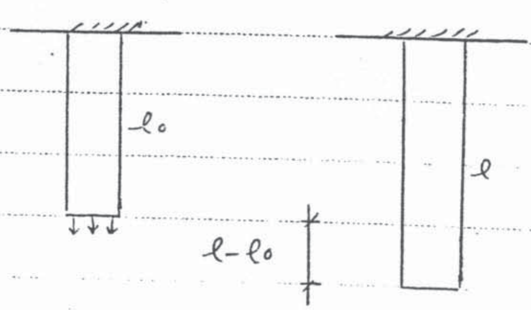
کرنش:

تغییر مکان به دو نوع میتواند باشد: جابجایی یا چرخش - تغییر شکل نسبی.

تغییر شکل محوری یا عمودی ، تغییر شکل زاویه ای یا برشی (لغزش).



مثال: (کتاب دکتر سعادت پور)



$$\varepsilon_1 = \frac{l-l_0}{l_0} \qquad \varepsilon_2 = \frac{l-l_0}{l}$$

تعاریف مختلفی از کرنش که میتوان متصور بود.

$$\varepsilon_3 = \frac{l^2-l_0^2}{2l_0^2} \qquad \varepsilon_4 = \frac{l^2-l_0^2}{2l^2}$$

$l_0 = 1 \qquad l = 1.25 \qquad\qquad l_0 = 1 \qquad l = 1.025$

$\hookrightarrow \varepsilon_1 = 0.2500 \qquad\qquad \hookrightarrow \varepsilon_1 = 0.0250$

$\varepsilon_2 = 0.2000 \qquad\qquad \varepsilon_2 = 0.0244$

$\varepsilon_3 = 0.2812 \qquad\qquad \varepsilon_3 = 0.02531$

$\varepsilon_4 = 0.1800 \qquad\qquad \varepsilon_4 = 0.02409$

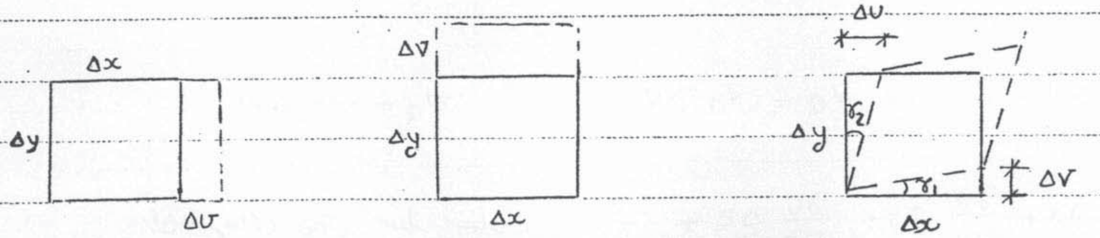
* در مسائل مورد بررسی ما چون اجسام تحت تنش هستند و کرنش ها کم هستند تعریف کرنش چندان اهمیت پیدا نمیکند.

$$\varepsilon = \frac{(l+\delta l) - l}{l} = \frac{\delta l}{l}$$

کرنش واقعی :

$$\varepsilon = \sum_{l_0}^{l} \frac{dl}{l} \rightsquigarrow \varepsilon = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0}$$

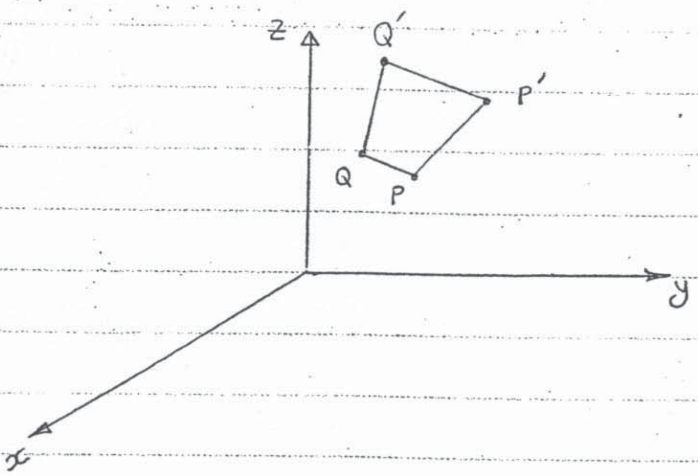
تعریف کرنش در این درس :



$$* \ \varepsilon_{xx} = \frac{\Delta u}{\Delta x} \qquad * \ \varepsilon_{yy} = \frac{\Delta v}{\Delta y} \qquad \gamma_{xy} = \gamma_1 + \gamma_2 \qquad \begin{cases} \tan\gamma_1 = \dfrac{\Delta v}{\Delta x} \\ \tan\gamma_2 = \dfrac{\Delta u}{\Delta y} \end{cases}$$

$$* \ \gamma_{xy} = \frac{\Delta v}{\Delta x} + \frac{\Delta u}{\Delta y}$$

تا اینجا کرنش سطحی مورد بحث قرار گرفت.



$$P\left|\begin{matrix} x \\ y \\ z \end{matrix}\right. \qquad P'\left|\begin{matrix} x' \\ y' \\ z' \end{matrix}\right.$$

$$\Omega = iU + jV + KW$$

$$Q\left|\begin{matrix} x+\Delta x \\ y+\Delta y \\ z+\Delta z \end{matrix}\right.$$

$$u = U(x, y, z) \qquad V = V(x, y, z) \qquad W = W(x, y, z)$$

$$P'\left|\begin{matrix} x' = x + U \\ y' = y + V \\ z' = z + W \end{matrix}\right.$$

$$u = u(x+\Delta x, y+\Delta y, z+\Delta z) \qquad V = V(x+\Delta x, y+\Delta y, z+\Delta z)$$

$W = \ldots$

$$u_Q = u + \Delta U \qquad V_Q = V + \Delta V \qquad W_Q = W + \Delta W$$

$$\Delta U = \frac{\partial U}{\partial x}\Delta x + \frac{\partial U}{\partial y}\Delta y + \frac{\partial U}{\partial z}\Delta z + \cdots$$

$$\Delta V = \frac{\partial V}{\partial x}\Delta x + \frac{\partial V}{\partial y}\Delta y + \frac{\partial V}{\partial z}\Delta z + \cdots$$

$$\Delta W = \frac{\partial W}{\partial x}\Delta x + \frac{\partial W}{\partial y}\Delta y + \frac{\partial W}{\partial z}\Delta z + \cdots$$

توان دوم های (به بعد) مقدار بسیار
کوچکی هستند و از آنها صرفنظر
میتوان کرد.

$$u_Q = u + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z$$

$$v_Q = v + \frac{\partial v}{\partial x}\Delta x + \frac{\partial v}{\partial y}\Delta y + \frac{\partial v}{\partial z}\Delta z$$

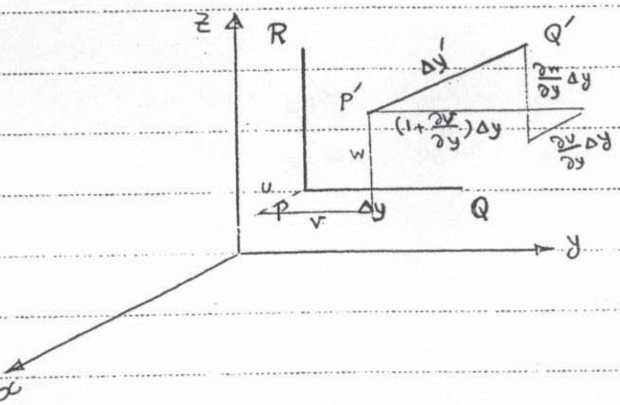
$$w_Q = w + \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \frac{\partial w}{\partial z}\Delta z$$

$$Q' \left| \begin{array}{l} x+\Delta x+u_Q \\ y+\Delta y+v_Q \\ z+\Delta z+w_Q \end{array} \right.$$

گرادیان دیپلسمان:

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix} = \left[\frac{\partial u_i}{\partial x_j}\right]$$

با داشتن این گرادیان در هر نقطه میتوان کرنش در نقطه را بدست آورد.



$$u_Q = u + \frac{\partial u}{\partial y}\Delta y$$

$$v_Q = v + \frac{\partial v}{\partial y}\Delta y$$

$$w_Q = w + \frac{\partial w}{\partial y}\Delta y$$

$$\varepsilon_{yy} = \frac{\Delta y' - \Delta y}{\Delta y} \qquad (\Delta y')^2 = (1+\varepsilon_{yy})^2(\Delta y)^2 \qquad \frac{(\Delta y')^2}{(\Delta y)^2} = (1+\varepsilon_{yy})^2$$

$$(\Delta y')^2 = \left[\left(1+\frac{\partial v}{\partial y}\right)\Delta y\right]^2 + \left[\frac{\partial u}{\partial y}\Delta y\right]^2 + \left[\frac{\partial w}{\partial y}\Delta y\right]^2$$

$$(1+\varepsilon_{yy})^2 = 1 + 2\frac{\partial v}{\partial y} + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2$$

$$\varepsilon_{yy} = \left[1 + 2\frac{\partial v}{\partial y} + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]^{1/2} - 1$$

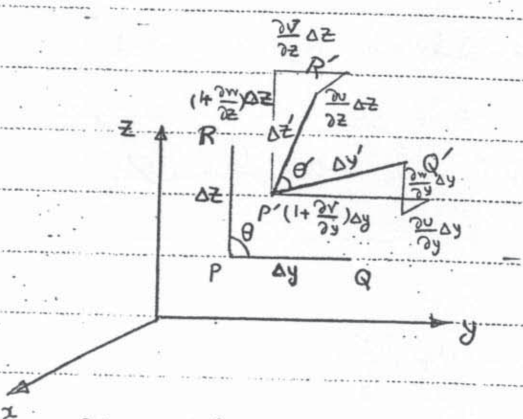
* روابط برای کرنش های خطی:

$$\therefore \quad \varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right]$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]$$

روابط دیگری برای کرنش برشی نیز در اینجا آورده میشود:



PQ	l_{xQ}	l_{yQ}	l_{zQ}	$P'Q'$	l'_{xQ}	l'_{yQ}	l'_{zQ}
PR	l_{xR}	l_{yR}	l_{zR}	$P'R'$	l'_{xR}	l'_{yR}	l'_{zR}

$$\cos\theta' = \sin\left(\frac{\pi}{2} - \theta'\right)$$

$$\cos\theta' = l'_{xQ}\, l'_{xR} + l'_{yQ}\, l'_{yR} + l'_{zQ}\, l'_{zR}$$

$$\cos\theta' = \left(\frac{\partial u}{\partial y} \times \frac{\Delta y}{\Delta y'}\right)\left(\frac{\partial u}{\partial z} \times \frac{\Delta z}{\Delta z'}\right) + \left[\left(1 + \frac{\partial v}{\partial y}\right)\frac{\Delta y}{\Delta y'}\right]\left(\frac{\partial v}{\partial z} \times \frac{\Delta z}{\Delta z'}\right) + \left(\frac{\partial w}{\partial y} \times \frac{\Delta y}{\Delta y'}\right)\left[\left(1 + \frac{\partial w}{\partial z}\right)\frac{\Delta z}{\Delta z'}\right]$$

$$\sin \gamma_{yz} = \left(\frac{\Delta y}{\Delta y'} \times \frac{\Delta z}{\Delta z'}\right)\left[\frac{\partial u}{\partial y} \times \frac{\partial u}{\partial z} + \left(1+\frac{\partial v}{\partial y}\right)\frac{\partial v}{\partial z} + \left(1+\frac{\partial w}{\partial z}\right)\frac{\partial w}{\partial y}\right]$$

$$\frac{\Delta z'}{\Delta z} = (1+\varepsilon_{zz}) \qquad \frac{\Delta y'}{\Delta y} = (1+\varepsilon_{yy})$$

$$* \ \gamma_{yz} = \arcsin \frac{1}{(1+\varepsilon_{yy})(1+\varepsilon_{zz})}\left(\frac{\partial u}{\partial z} \times \frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} \times \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \times \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$* \ \gamma_{zx} = \arcsin \frac{1}{(1+\varepsilon_{zz})(1+\varepsilon_{xx})}\left(\frac{\partial u}{\partial z} \times \frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} \times \frac{\partial v}{\partial x} + \frac{\partial w}{\partial z} \times \frac{\partial w}{\partial x} + \frac{\partial w}{\partial x} + \frac{\partial v}{\partial z}\right)$$ روابط برای کرنش زاویه ای

$$* \ \gamma_{xy} = \arcsin \frac{1}{(1+\varepsilon_{xx})(1+\varepsilon_{yy})}\left(\frac{\partial u}{\partial x} \times \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \times \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x} \times \frac{\partial w}{\partial y} + \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

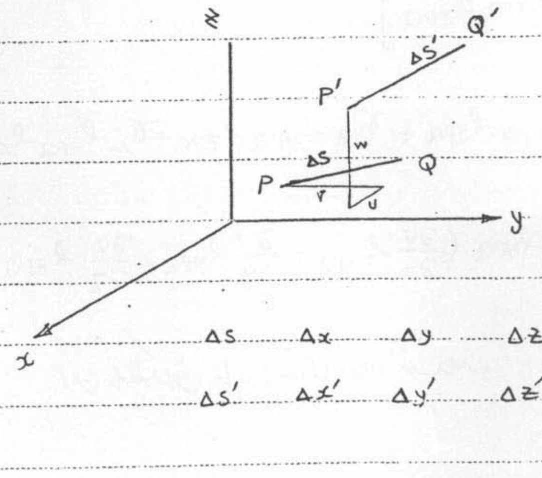
در کرنش مهندسی این روابط را ساده تر میکنیم.

* روابط کاربردی در مهندسی:

$$* \ \varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad * \ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$* \ \varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad * \ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$* \ \varepsilon_{zz} = \frac{\partial w}{\partial z} \qquad * \ \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$



$$(\Delta S)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

$$(\Delta S')^2 = (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2$$

$\Delta S \quad \Delta x \quad \Delta y \quad \Delta z$

$\Delta S' \quad \Delta x' \quad \Delta y' \quad \Delta z'$

$$\Delta x' = \Delta x + \Delta u$$

$$\Delta x' = \left(1+\frac{\partial u}{\partial x}\right)\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z$$

$$\Delta y' = \frac{\partial v}{\partial x}\Delta x + \left(1+\frac{\partial v}{\partial y}\right)\Delta y + \frac{\partial u}{\partial z}\Delta z$$

$$\Delta z' = \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \left(1+\frac{\partial w}{\partial z}\right)\Delta z$$

$$(\Delta x')^2 = \left(1+\frac{\partial u}{\partial x}\right)^2(\Delta x)^2 + \left[\frac{\partial u}{\partial y}\Delta y\right]^2 + \left[\frac{\partial u}{\partial z}\Delta z\right]^2 + 2\left[\left(1+\frac{\partial u}{\partial x}\right)\Delta x\frac{\partial u}{\partial y}\Delta y\right] +$$

$$2\left[1+\frac{\partial u}{\partial x}\Delta x\frac{\partial u}{\partial z}\Delta z\right] + 2\frac{\partial u}{\partial y}\Delta y\frac{\partial u}{\partial z}\Delta z$$

$$= (\Delta x)^2 + \left(\frac{\partial u}{\partial x}\right)^2\Delta x^2 + 2\frac{\partial u}{\partial x}\Delta x^2 + \cdots$$

$$\hookrightarrow (\Delta x')^2 = (\Delta x)^2 + 2\frac{\partial u}{\partial x}(\Delta x)^2 + 2\frac{\partial u}{\partial y}\Delta y\Delta x + 2\frac{\partial u}{\partial z}\Delta z\Delta x$$

$$(\Delta s')^2 = \left(1+2\frac{\partial u}{\partial x}\right)\Delta x^2 + \left(1+2\frac{\partial v}{\partial y}\right)\Delta y^2 + \left(1+2\frac{\partial w}{\partial z}\right)\Delta z^2 + 2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\Delta x\Delta y$$

$$+ 2\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\Delta y\Delta z + 2\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\Delta z\Delta x$$

$$\varepsilon_{PQ} = \frac{\Delta s' - \Delta s}{\Delta s} \qquad (\varepsilon_{PQ}+1)^2 = \frac{(\Delta s')^2}{(\Delta s)^2}$$

$$l_{xPQ} = \frac{\Delta x}{\Delta s} \qquad l_{yPQ} = \frac{\Delta y}{\Delta s} \qquad l_{zPQ} = \frac{\Delta z}{\Delta s}$$

$$l_{xPQ}^2 + l_{yPQ}^2 + l_{zPQ}^2 = 1$$

$$(\varepsilon_{PQ}+1)^2 = 1 + 2\left[\frac{\partial u}{\partial x}l_{xPQ}^2 + \frac{\partial v}{\partial y}l_{yPQ}^2 + \frac{\partial w}{\partial z}l_{zPQ}^2 + \left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)l_{xPQ}l_{yPQ}\right.$$

$$\left. + \left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)l_{yPQ}\,l_{zPQ} + \left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)l_{xPQ}l_{zPQ}\right]$$

برای $* \ \varepsilon_{PQ} = \varepsilon_{xx}l_{xPQ}^2 + \varepsilon_{yy}l_{yPQ}^2 + \varepsilon_{zz}l_{zPQ}^2 + \gamma_{xy}l_{xPQ}l_{yPQ} + \gamma_{yz}l_{yPQ}l_{zPQ} + \gamma_{zx}l_{zPQ}l_{xPQ}$

برای $* \ \varepsilon_{PQ} = l_{xPQ}\left(\frac{\partial u}{\partial x}l_{xPQ} + \frac{\partial u}{\partial y}l_{yPQ} + \frac{\partial u}{\partial z}l_{zPQ}\right) + l_{yPQ}\left(\frac{\partial v}{\partial x}l_{xPQ} + \frac{\partial v}{\partial y}l_{yPQ} + \frac{\partial v}{\partial z}l_{zPQ}\right) +$
های

$$l_{zPQ}\left(\frac{\partial w}{\partial x}l_{xPQ} + \frac{\partial w}{\partial y}l_{yPQ} + \frac{\partial w}{\partial z}l_{zPQ}\right)$$

این رابطه کرنش را در راستای PQ میدهد.

۸،۸ تمرین : مقادیر تغییر شکل نسبی را در دستگاه $x''y''z''$ بیان نمایید . (با فرض داشتن تغییر شکل نسبی در دستگاه xyz)

(حل را ننوشتیم زاویه‌ای بصورت تمرین ارائه شد) با الفبای زاویه‌ای از کتاب یادداشت شود.

$$\varepsilon_{x'x'} = \varepsilon_{xx} l^2_{xx'} + \varepsilon_{yy} l^2_{yx'} + \varepsilon_{zz} l^2_{zx'} + \gamma_{xy} l_{xx'} l_{yx'} + \gamma_{yz} l_{yx'} l_{zx'} + \gamma_{zx} l_{zx'} l_{xx'}$$

$$\varepsilon_{y'y'} = \cdots$$

$$\varepsilon_{z'z'} = \cdots$$

$$\gamma_{x'y'} = 2\varepsilon_{xx} l_{xx'} l_{xy'} + 2\varepsilon_{yy} l_{yx'} l_{yy'} + 2\varepsilon_{zz} l_{zx'} l_{zy'} + \gamma_{xy} (l_{xx'} l_{yy'} + l_{xy'} l_{yx'})$$

$$+ \gamma_{yz} (l_{yx'} l_{zy'} + l_{yy'} l_{zx'}) + \gamma_{zx} (l_{zx'} l_{xy'} + l_{yy'} l_{xx'})$$

* اصلاح 2 ← و ایراد گرفته شود

$$\gamma_{y'z'} = \cdots$$

$$\gamma_{z'x'} = \cdots$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \qquad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial y}$$

فرم اندیسی :

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \qquad i, j = x, y, z$$

$$u_x = u$$

$$u_y = v \qquad \varepsilon_{xy} = \frac{1}{2} \gamma_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

* روابط مهم (مطابق با کتاب سعادت‌پور)

$$x_x = x$$

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تانسور کرنش :

$$\begin{bmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{yx} & \frac{1}{2}\gamma_{zx} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_{yy} & \frac{1}{2}\gamma_{zy} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_{zz} \end{bmatrix}$$

* یافتن کرنش‌ها :

$$\varepsilon_{PQ} = \varepsilon_{xx}\, l_{xPQ}^2 + \varepsilon_{yy}\, l_{yPQ}^2 + \varepsilon_{zz}\, l_{zPQ}^2 + \gamma_{xy}\, l_{xPQ}\, l_{yPQ} + \gamma_{yz}\, l_{yPQ}\, l_{zPQ} + \gamma_{zx}\, l_{zPQ}\, l_{xPQ}$$

ضمناً داریم ، $l_{xPQ}^2 + l_{yPQ}^2 + l_{zPQ}^2 = 1$

$$\frac{\partial \varepsilon_{PQ}}{\partial l_{xPQ}} = 0 \qquad \frac{\partial \varepsilon_{PQ}}{\partial l_{yPQ}} = 0 \qquad \frac{\partial \varepsilon_{PQ}}{\partial l_{zPQ}} = 0$$

← حل به روش ضریب لاگرانژ : (ابتدا روش ریاضی را توضیح میدهیم)

یک تابع داریم : $F(x,y,z)$

یک شرط داریم : $g(x,y,z) = 0$

حال مقدار حدی $F(x,y,z)$ را می‌خواهیم.

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}\left(\frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy\right)$$

$$\Rightarrow dF = \left(\frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\times\frac{\partial z}{\partial x}\right)dx + \left(\frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}\times\frac{\partial z}{\partial y}\right)dy = 0$$

$$\begin{cases} \dfrac{\partial F}{\partial x} + \dfrac{\partial F}{\partial z}\times\dfrac{\partial z}{\partial x} = 0 \\[2ex] \dfrac{\partial F}{\partial y} + \dfrac{\partial F}{\partial z}\times\dfrac{\partial z}{\partial y} = 0 \end{cases}$$

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برای تابع g هم داریم :

$$\begin{cases} \dfrac{\partial g}{\partial x} + \dfrac{\partial g}{\partial z} \times \dfrac{\partial z}{\partial x} = 0 \\[2ex] \dfrac{\partial g}{\partial y} + \dfrac{\partial g}{\partial z} \times \dfrac{\partial z}{\partial y} = 0 \end{cases} \qquad \underline{\underline{2}}$$

با جایگزینی ۲ در ۱ داریم :

$$* \; \frac{\partial z}{\partial x} = -\frac{\partial g/\partial x}{\partial g/\partial z} \qquad * \; \frac{\partial z}{\partial y} = -\frac{\partial g/\partial y}{\partial g/\partial z}$$

$$\frac{\partial F}{\partial x} - \frac{\partial F/\partial z}{\partial g/\partial z} \times \frac{\partial g}{\partial x} = 0$$

$$\frac{\partial F}{\partial y} - \frac{\partial F/\partial z}{\partial g/\partial z} \times \frac{\partial g}{\partial y} = 0$$

$$\lambda = \frac{\partial F/\partial z}{\partial g/\partial z}$$

$$\frac{\partial F}{\partial x} - \lambda \frac{\partial g}{\partial x} = 0 \qquad \frac{\partial F}{\partial y} - \lambda \frac{\partial g}{\partial y} = 0 \qquad \frac{\partial F}{\partial z} - \lambda \frac{\partial g}{\partial z} = 0$$

اپراتور لاگرانژ :

$$\underline{U(x,y,z) = F(x,y,z) - \lambda\, g(x,y,z)}$$

در این داریم :

$$F(x,y,z) \longrightarrow F(l_{xPQ}, l_{yPQ}, l_{zPQ})$$

تابع شرطی

$$g \longrightarrow g(l_{xPQ}, l_{yPQ}, l_{zPQ}) \longrightarrow l_{xPQ}^2 + l_{yPQ}^2 + l_{zPQ}^2 - 1 = 0$$

معادلات روابط لاگرانژ را بازنویسی می کنیم:

$$\begin{cases} \dfrac{\partial \varepsilon_{PQ}}{\partial l_{xPQ}} + \dfrac{\partial \varepsilon_{PQ}}{\partial l_{zPQ}} \times \dfrac{\partial z_{PQ}}{\partial l_{xPQ}} = 0 \\ \dfrac{\partial \varepsilon_{PQ}}{\partial l_{yPQ}} + \dfrac{\partial \varepsilon_{PQ}}{\partial l_{zPQ}} \times \dfrac{\partial l_{zPQ}}{\partial l_{yPQ}} = 0 \end{cases}$$

$$\begin{cases} 2\, l_{xPQ} + 2\, l_{zPQ} \times \dfrac{\partial l_{zPQ}}{\partial l_{xPQ}} = 0 \\ 2\, l_{yPQ} + 2\, l_{zPQ} \times \dfrac{\partial l_{zPQ}}{\partial l_{yPQ}} = 0 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \dfrac{\partial l_{zPQ}}{\partial l_{xPQ}} = \dfrac{-2 l_{xPQ}}{2 l_{zPQ}} \\ \dfrac{\partial l_{zPQ}}{\partial l_{yPQ}} = \dfrac{-2 l_{yPQ}}{2 l_{zPQ}} \end{cases}$$

$$\hookrightarrow \begin{cases} \dfrac{\partial \varepsilon_{PQ}}{\partial l_{xPQ}} - \dfrac{\partial \varepsilon_{PQ}}{\partial l_{zPQ}} \times \dfrac{l_{xPQ}}{l_{zPQ}} = 0 \\ \dfrac{\partial \varepsilon_{PQ}}{\partial l_{yPQ}} - \dfrac{\partial \varepsilon_{PQ}}{\partial l_{zPQ}} \times \dfrac{l_{yPQ}}{l_{zPQ}} = 0 \end{cases}$$

$$\begin{cases} \dfrac{\partial \varepsilon_{PQ}}{\partial l_{xPQ}} - \dfrac{\partial \varepsilon_{PQ} / \partial l_{zPQ}}{2 l_{zPQ}} \times 2\, l_{xPQ} = 0 \\ \dfrac{\partial \varepsilon_{PQ}}{\partial l_{yPQ}} - \dfrac{\partial \varepsilon_{PQ} / \partial l_{zPQ}}{2 l_{zPQ}} \times 2\, l_{yPQ} = 0 \end{cases}$$

$$\lambda = \varepsilon = \frac{\partial \varepsilon_{PQ} / \partial l_{zPQ}}{2 l_{zPQ}} \Rightarrow$$

$$\varepsilon = \frac{2 \varepsilon_{zz} l_{zPQ} + \gamma_{yz} l_{yPQ} + \gamma_{zx} l_{xPQ}}{2 l_{zPQ}} \rightarrow$$

با طرفین وسطین داریم

$$\begin{cases} 2 \varepsilon_{xx} l_{xPQ} + \gamma_{xy} l_{yPQ} + \gamma_{zx} l_{zPQ} - 2 \varepsilon\, l_{xPQ} = 0 \\ 2 \varepsilon_{yy} l_{yPQ} + \gamma_{xy} l_{xPQ} + \gamma_{yz} l_{zPQ} - 2 \varepsilon\, l_{yPQ} = 0 \\ 2 \varepsilon_{zz} l_{zPQ} + \gamma_{xz} l_{xPQ} + \gamma_{yz} l_{yPQ} - 2 \varepsilon\, l_{zPQ} = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 2(\varepsilon_{xx}-\varepsilon)\, l_{xPQ} + \gamma_{xy}\, l_{yPQ} + \gamma_{zx}\, l_{zPQ} = 0 \\ \gamma_{xy}\, l_{xPQ} + 2(\varepsilon_{yy}-\varepsilon)\, l_{yPQ} + \gamma_{yz}\, l_{zPQ} = 0 \\ \gamma_{xz}\, l_{xPQ} + \gamma_{yz}\, l_{yPQ} + 2(\varepsilon_{zz}-\varepsilon)\, l_{zPQ} = 0 \end{cases}$$

$$\varepsilon^3 - J_1 \varepsilon^2 + J_2 \varepsilon - J_3 = 0$$

انواریان های کرنش :   * یادآوری گردد (از بحث [illegible])

$$J_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

$$J_2 = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{vmatrix} + \begin{vmatrix} \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{yz} & \varepsilon_{zz} \end{vmatrix} + \begin{vmatrix} \varepsilon_{zz} & \varepsilon_{xz} \\ \varepsilon_{xz} & \varepsilon_{xx} \end{vmatrix}$$

$$J_3 = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{vmatrix}$$

or :

$$J_2 = -\varepsilon_{xy}^2 - \varepsilon_{yz}^2 - \varepsilon_{zx}^2 + \varepsilon_{xx}\varepsilon_{yy} + \varepsilon_{yy}\varepsilon_{zz} + \varepsilon_{zz}\varepsilon_{xx}$$

$$J_3 = \varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz} + 2\varepsilon_{xy}\varepsilon_{yz}\varepsilon_{zx} - \varepsilon_{xx}\varepsilon_{yz}^2 - \varepsilon_{yy}\varepsilon_{zx}^2 - \varepsilon_{zz}\varepsilon_{xy}^2$$

نوع دیگر نگارش : در مختصات اصلی   در مختصات اصلی :

$$J_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

$$J_2 = -\varepsilon_1\varepsilon_2 - \varepsilon_2\varepsilon_3 - \varepsilon_3\varepsilon_1$$ چرا منفی ؟

$$J_3 = \varepsilon_1\varepsilon_2\varepsilon_3$$

* اگر در یک نقطه تنش ها را داشته باشیم میتوانیم کرنش ها را بدست آوریم. اما با داشتن کرنش ها در یک نقطه نمیتوان به تنش ها رسید.

نیاز به یک فیلتر (معادلات سازگاری) داریم تا از کرنش به تنش برسیم.

← یعنی نمیتوان هر تانسور کرنشی را به اختیار نوشت. باید با توابع سازگاری، مطابق داشته باشد.

بدست آوردن توابع سازگاری:

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} \qquad 2\frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 \gamma_{yz}}{\partial y \partial z} = \frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} \qquad 2\frac{\partial^2 \varepsilon_{yy}}{\partial z \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 \gamma_{zx}}{\partial z \partial x} = \frac{\partial^2 \varepsilon_{zz}}{\partial x^2} + \frac{\partial^2 \varepsilon_{xx}}{\partial z^2} \qquad 2\frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right)$$

(سه رابطه مستقل داریم) "سن ونان" Saint Venant.

هم بند ساده: Simply Connected Body

تمرین: هم بند ساده بودن روابط بالا را بررسی کنید.

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$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial^3 v}{\partial x^2 \partial y}$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} = \frac{\partial^3 u}{\partial x \partial y^2} \qquad \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^3 v}{\partial x^2 \partial y}$$

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2}$$ درحالت دوبعدی

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \qquad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial z} = \frac{\partial^3 u}{\partial x \partial y \partial z} + \frac{\partial^3 v}{\partial x^2 \partial z} \qquad (1)$$

$$\frac{\partial^2 \gamma_{zx}}{\partial x \partial y} = \frac{\partial^3 w}{\partial x^2 \partial y} + \frac{\partial^3 u}{\partial x \partial y \partial z} \qquad (2)$$

$$\frac{\partial \gamma_{zy}}{\partial x^2} = \frac{\partial^3 v}{\partial x^2 \partial z} + \frac{\partial^3 w}{\partial x^2 \partial y} \qquad (3)$$

(1) + (2) − (3) →

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial z} + \frac{\partial^2 \gamma_{zx}}{\partial x \partial y} - \frac{\partial^2 \gamma_{yz}}{\partial x^2} = 2 \frac{\partial^3 u}{\partial x \partial y \partial z}$$

$$\frac{\partial^2 \varepsilon_{xy}}{\partial y \partial z} = \frac{\partial^3 u}{\partial x \partial y \partial z}$$

$$\Rightarrow 2 \frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z} = \frac{\partial}{\partial x} \left(\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \gamma_{yz}}{\partial x} \right)$$

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مثال : $\varepsilon_{xx} = 3x^2y \qquad \varepsilon_{yy} = 4y^2x \qquad \varepsilon_{xy} = yx + x^3$

کنترل روابط سازگاری :

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$ (رابطه صفحه‌ای)

$$\frac{\partial \varepsilon_{xx}}{\partial y} = 3x^2 \qquad \frac{\partial \varepsilon_{yy}}{\partial x} = 4y^2 \qquad \frac{\partial \varepsilon_{xy}}{\partial x} = y - 3x^2$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} = 0 \qquad \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = 0 \qquad \frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} = 1$$

$0 + 0 = 2 \times 1$ ✗

$$\varepsilon_{zz} = \gamma_{xz} = \gamma_{yz} = 0$$

$$\varepsilon_{xx} = K(x^2 + y^2) \qquad \varepsilon_{yy} = K(y^2 + z^2) \qquad \gamma_{xy} = K'xyz$$

$$2K = K'z$$

$$\frac{\partial \varepsilon_{xx}}{\partial y} = 2Ky \qquad \frac{\partial \varepsilon_{yy}}{\partial x} = 0 \qquad \frac{\partial \gamma_{xy}}{\partial x} = K'yz$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} = 2K \qquad \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = 0 \qquad \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = K'z$$

* مکانیزم ها و روش های گوناگونی در تشخیص و اندازه گیری ترک وجود دارد که تمامی آنها با هدف یافتن ترک و

متعاقب آن تنش مورد استفاده قرار می گیرند.

* بطور کلی کمتر علاقمند به یافتن ترک بطور صرف هستیم. معیار کنترل تنش در این سازه ها (در اکثر موارد)

معیار کنترلی میباشد. بنابراین هدف نهایی در اکثر موارد یافتن تنش میباشد.

فوتو الاستیسیته: روشی که در آن نور پولاریزه را از جسم شفاف عبور میدهند و توزیع تنش را در جسم مورد بررسی

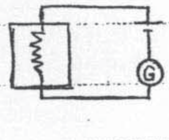
قرار میدهند.

* یک روش دیگر بررسی وضعیت ترک در جسم، است که روی جسم رنگ شکننده ورقه بزنیم و سپس جسم

را تحت بارگذاری بررسی کنیم.

* روش دیگری که میزان مشهور بود است که از خاصیت تغییر مقاومت یک جسم بدلیل تغییر سطح مقطع آن در اثر

تغییر طول استفاده کنیم. $R = \rho \frac{L}{A}$ مقاومت الکتریکی (طول: L، سطح مقطع: A)



← کرنش سنج های الکتریکی .. Strain Gauge.

* سیستم استرین گیج در صنعت ساختمان و مهندسی عمران نسبت به روش های دیگر اندازه گیری کرنش کاربرد بیشتری دارد.

اندازه گیری تغییرات مقاومت استرین گیج با استفاده از پل وستون است.

انواع آرایش Strain gage ها: