سری پنجم

① $\to \frac{1}{n}\left(\frac{1}{(1+\frac{1}{n})^{۲}}+\frac{1}{(1+\frac{۲}{n})^{۲}}+\dots+\frac{1}{(1+\frac{n}{n})^{۲}}\right) \to \int_0^1 \frac{dx}{(1+x)^{۲}}$

$\to = -\frac{1}{۲}(1+x)^{-۲}\Big|_0^1 = \frac{1}{۲}\left(1-\frac{1}{۴}\right) = \underline{\underline{\frac{۳}{۸}}}$

لم: $\frac{1}{n}\sum_{1}^{n} f\left(\frac{k}{n}\right) \to \int_0^1 f(x)\,dx$

[illegible] $\frac{1}{n+1}+\frac{1}{n+۲}+\dots+\frac{1}{۲n} \to \frac{1}{n}\left(\frac{1}{1+\frac{1}{n}}+\dots+\frac{1}{1+\frac{n}{n}}\right) \to \int_0^1 \frac{dx}{1+x} = \underline{\ln ۲}$

[illegible] $\begin{cases} \frac{f(x)}{g(x)} \to 1 \\ f-g \to 0 \end{cases}$ $, \quad fh \simeq gh \Rightarrow fh = \frac{f}{g}hg = \underline{gh}$

تابع هم‌ارز $\sin x \sim x \to \frac{\sin x}{x} \to 1 \ , \ \sin x - x \to 0$

$\cos x \sim 1-\frac{x^{۲}}{۲} \to \frac{\cos x}{1-\frac{x^{۲}}{۲}} \to 1 \ , \ \cos x + \frac{x^{۲}}{۲} - 1 \to 0$

مثال، $1, \frac{1}{۲}, \frac{1}{۳}, \dots, \frac{1}{n} \to \frac{1}{n}\left(\frac{1}{\frac{1}{n}}+\frac{1}{\frac{۲}{n}}+\dots+\frac{1}{\frac{n}{n}}\right)$

$\to \int_0^1 \frac{1}{x}\,dx = \ln x\Big|_0^1 = \underline{\infty}$

② مقدار میانگین $\to \int_a^b f-g = 0 = (b-a)(f-g)_{(c)} \to (f-g)(c) = 0$

$\to f(c) = g(c)$

③ [illegible] ام $\int g = 1 \to$ [illegible] $\left(\frac{1}{۲}p_1 + \frac{1}{۴}p_۲ + \frac{1}{۴}p_۳\right)$

$\int g \neq 1 \to h(x) = \frac{g(x)}{\int g} \Rightarrow \int h = 1 \to \int f(x)h(x)\,dx = f(c)$

$$m < f < M \to m < \frac{\int fg}{\int g} < M \to m < f(c) < M$$ ادامه

مثال

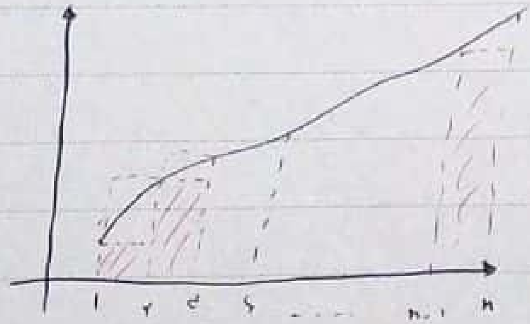
$$m \le f \le M \to m^2 = \int_0^1 m^2 \le \left(\int_0^1 f^2\right)^{\frac{1}{2}} \le \int_0^1 M^2 = M^2$$

$$\Rightarrow \left(\int_0^1 f^2\right)^{\frac{1}{2}} = f(c)$$

$$m \le f \le M \to e^m \le e^f \le e^M \to m \le \ln\int_0^1 e^f \le M$$

$$\Rightarrow \ln\int_0^1 e^f = f(c)$$

۴) الف



$$\to f(1)\times 1 + f(2)\times 1 + \dots + f(n-1)\times 1 \le \int_1^n f(x)\,dx \le f(2)\times 1 + f(3)\times 1 + \dots + f(n)\times 1$$

$$\to \sum_{k=1}^{n-1} f(k) \le \int_1^n f(x)\,dx \le \sum_{k=2}^{n} f(k)$$

مثال

$$f(x) = \frac{1}{x}$$

$$\to \frac{1}{2} + \dots + \frac{1}{n} \le \underbrace{\int_1^n \frac{dx}{x}}_{\ln n} \le 1 + \frac{1}{2} + \dots + \frac{1}{n-1} \to \underbrace{\frac{1}{n} + \overset{1}{\Uparrow}\ln n}_{\ln n} \le \underbrace{1 + \frac{1}{2} + \dots + \frac{1}{n}}_{\ln n} \le \underbrace{\overset{1}{\Uparrow}1 + \ln n}_{\ln n}$$

یعنی تقریبا ۱ ... تقریبا مرتبه لن

مرتبه

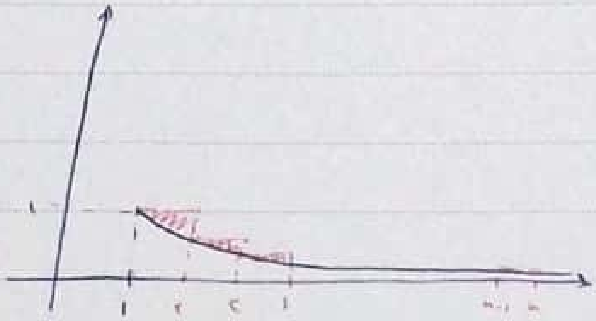
$$\Rightarrow 1 + \frac{1}{2} + \dots + \frac{1}{n} \simeq \underline{\underline{\ln n}}$$

کران دار است

$$\frac{1}{n} \le 1 + \frac{1}{2} + \dots + \frac{1}{n} - \ln n \le 1$$

حفظ شد

$e_n = 1+\frac{1}{۲}+\frac{1}{۳}+\dots+\frac{1}{n}-\ln n \to ?$



$\to 0 \le e_n = \underbrace{1+\frac{1}{۲}+\frac{1}{۳}+\dots+\frac{1}{n}-\ln n}_{?} + \underset{\downarrow\,0}{\frac{1}{n}} \le 1$

اویلر γ ← مقدار ← = [illegible]

تمرین: $1+\frac{1}{۲^\alpha}+\dots+\frac{1}{n^\alpha}$

ب) $\ln 1+\ln ۲+\dots+\ln(n-1) \le \int_1^n \ln x\,dx \le \ln ۲+\dots+\ln n$

$\to \ln(n-1)! \le n\ln n-n+1 \le \ln n!$

$\to (n-1)! \le n^n \cdot e^{-n+1} \le n!$

$\to n^n e^{-n+1} \le n! \le (n+1)^{n+1} e^{-n}$