روش های عددی حل رسانش پایا دو بعدی

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\dot q}{k}=0$$

$$\left.\frac{\partial T}{\partial x}\right]_{m+\frac12,n}=\frac{T_{m+1,n}-T_{m,n}}{\Delta x}\quad,\quad \left.\frac{\partial T}{\partial x}\right]_{m-\frac12,n}=\frac{T_{m,n}-T_{m-1,n}}{\Delta x}$$

$$\left.\frac{\partial^2 T}{\partial x^2}\right]_{m,n}=\frac{\left.\frac{\partial T}{\partial x}\right]_{m+\frac12,n}-\left.\frac{\partial T}{\partial x}\right]_{m-\frac12,n}}{\Delta x}=\frac{T_{m+1,n}+T_{m-1,n}-2T_{m,n}}{\Delta x^2}$$

به همین ترتیب

$$\left.\frac{\partial^2 T}{\partial y^2}\right]_{m,n}=\frac{T_{m,n+1}+T_{m,n-1}-2T_{m,n}}{\Delta y^2}$$

به طور کلی

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\dot q}{k}=0 \Rightarrow \frac{T_{m+1,n}+T_{m-1,n}-2T_{m,n}}{\Delta x^2}+\frac{T_{m,n-1}+T_{m,n+1}-2T_{m,n}}{\Delta y^2}+\frac{\dot q}{k}=0$$

روش عددی حل رسانش گذرا دو بعدی

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2}=\frac{T^{p}_{m+1,n}+T^{p}_{m-1,n}-2T^{p}_{m,n}}{\Delta x^2}\quad,\quad \frac{\partial^2 T}{\partial y^2}=\frac{T^{p}_{m,n+1}+T^{p}_{m,n-1}-2T^{p}_{m,n}}{\Delta y^2}$$

$$\frac{\partial T}{\partial t}=\frac{T^{p+1}_{m,n}-T^{p}_{m,n}}{\Delta t}$$

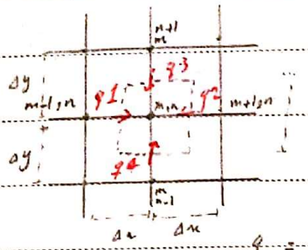
$$\Rightarrow \frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\dot q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}\Rightarrow$$

فرم کلی

$$\Rightarrow \frac{T^{p}_{m,n+1}+T^{p}_{m,n-1}-2T^{p}_{m,n}}{\Delta x^2}+\frac{T^{p}_{m,n+1}+T^{p}_{m,n-1}-2T^{p}_{m,n}}{\Delta y^2}+\frac{\dot q}{k}=\frac{1}{\alpha}\frac{T^{p+1}_{m,n}-T^{p}_{m,n}}{\Delta t}$$

موقعیت نزدیک

الف) گره داخلی



$$q = -kA\frac{\partial T}{\partial x}$$

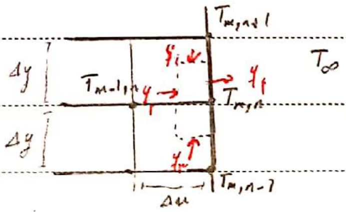
$$q_1 = -k\Delta y\frac{T_{m,n} - T_{m-1,n}}{\Delta x} \quad , \quad q_2 = -k\Delta y\frac{T_{m,n} - T_{m+1,n}}{\Delta x}$$

$$q_3 = -k\Delta x\frac{T_{m,n} - T_{m,n+1}}{\Delta y} \quad , \quad q_4 = -k\Delta x\frac{T_{m,n} - T_{m,n-1}}{\Delta y}$$

$$\sum q = 0 \;,\; \Delta x = \Delta y \Rightarrow 0 = T_{m,n} - T_{m-1,n} + T_{m,n} - T_{m+1,n} + T_{m,n} - T_{m,n+1} + T_{m,n} - T_{m,n-1}$$

$$\Rightarrow \boxed{T_{m,n} = \frac{1}{4}\left(T_{m-1,n} + T_{m+1,n} + T_{m,n+1} + T_{m,n-1}\right)}$$

ب) گره مرزی جابجایی



$$q_1 = -k\Delta y\frac{T_{m,n} - T_{m-1,n}}{\Delta x}$$

$$q_2 = -k\frac{\Delta x}{2}\,\frac{T_{m,n} - T_{m,n+1}}{\Delta y} \quad , \quad q_3 = -k\frac{\Delta x}{2}\,\frac{T_{m,n} - T_{m,n-1}}{\Delta y}$$

$$q_4 = -h\Delta y\left(T_\infty - T_{m,n}\right) \qquad \sum q = 0 \;,\; \Delta x = \Delta y \Rightarrow$$

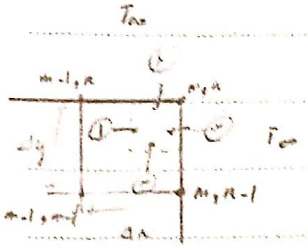
$$\Rightarrow -k\frac{\Delta y}{\Delta x}\left[T_{m,n} - T_{m-1,n} + \frac{T_{m,n}}{2} - \frac{T_{m,n+1}}{2} + \frac{T_{m,n}}{2} - \frac{T_{m,n-1}}{2}\right] = -h\Delta y\left(T_\infty - T_{m,n}\right)$$

$$2T_{m,n} - T_{m-1,n} - \frac{T_{m,n+1}}{2} - \frac{T_{m,n-1}}{2} - \frac{h\Delta y}{k}T_\infty + \frac{h\Delta y}{k}T_{m,n} = 0$$

$$\Rightarrow \left(2 + \frac{h\Delta y}{k}\right)T_{m,n} = T_{m-1,n} + \left(T_{m,n+1} + T_{m,n-1}\right)/2$$

$$\boxed{T_{m,n} = \frac{T_{m-1,n} + \left(T_{m,n+1} + T_{m,n-1}\right)/2}{2 + \frac{h\Delta y}{k}}}$$

ج) گوشه خارجی با مرز جابه جایی



$$q_1 = -k\frac{\Delta y}{2}\,\frac{T_{m,n}-T_{m-1,n}}{\Delta y}$$

$$q_2 = -k\frac{\Delta x}{2}\,\frac{T_{m,n}-T_{m,n-1}}{\Delta y} \quad , \quad q_3 = -h\frac{\Delta y}{2}\left(T_{m,n}-T_\infty\right)$$

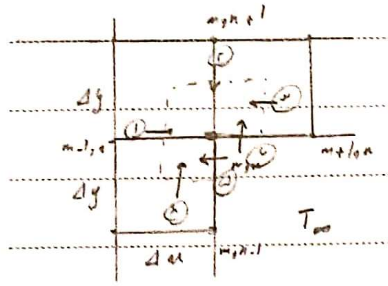
$$q_4 = -h\frac{\Delta x}{2}\left(T_{m,n}-T_\infty\right) \quad , \quad \Delta x = \Delta y \quad , \quad \sum q = 0 \quad \Rightarrow$$

$$\Rightarrow -\frac{k}{2}\left(T_{m,n}-T_{m-1,n}+T_{m,n}-T_{m,n-1}\right) - \frac{h\Delta x}{2}\left(T_{m,n}-T_\infty+T_{m,n}-T_\infty\right) = 0$$

$$2T_{m,n} - T_{m-1,n} - T_{m,n-1} + \frac{h\Delta x}{k}\,2T_{m,n} - \frac{h\Delta x}{k}\cdot 2T_\infty = 0$$

$$\boxed{T_{m,n} = \frac{T_{m-1,n}+T_{m,n-1}+\frac{2h\Delta x}{k}T_\infty}{2+\frac{2h\Delta x}{k}}}$$

د) گوشه داخلی با مرز جابه جایی



$$\Delta x = \Delta y \quad , \quad \sum q = 0$$

$$q_1 = -k\Delta y\,\frac{T_{m,n}-T_{m-1,n}}{\Delta x} \quad , \quad q_2 = -k\Delta x\,\frac{T_{m,n}-T_{m,n+1}}{\Delta y}$$

$$q_3 = -k\frac{\Delta y}{2}\,\frac{T_{m,n}-T_{m+1,n}}{\Delta x} \quad , \quad q_4 = -k\frac{\Delta x}{2}\,\frac{T_{m,n}-T_{m,n-1}}{\Delta y}$$

$$q_5 = -h\frac{\Delta y}{2}\left(T_{m,n}-T_\infty\right) \quad , \quad q_6 = -h\frac{\Delta x}{2}\left(T_{m,n}-T_\infty\right)$$

$$\sum q = -k\left(T_{m,n}-T_{m-1,n}+T_{m,n}-T_{m,n+1}+\frac{T_{m,n}}{2}-\frac{T_{m+1,n}}{2}+\frac{T_{m,n}}{2}-\frac{T_{m,n-1}}{2}\right) - h\Delta y\left(T_{m,n}-T_\infty\right)$$

$$3T_{m,n} - T_{m-1,n} - T_{m,n+1} - \frac{T_{m+1,n}}{2} - \frac{T_{m,n-1}}{2} + \frac{h\Delta y}{k}T_{m,n} - \frac{h\Delta y}{k}T_\infty = 0$$

$$\boxed{T_{m,n} = \frac{T_{m-1,n}+T_{m,n+1}+\frac{1}{2}\left(T_{m+1,n}+T_{m,n-1}\right)+\frac{h\Delta y}{k}T_\infty}{3+\frac{h\Delta y}{k}}}$$