$$\text{Thrust}\quad F = [(10)^2 + (4)^2]^{1/2}\left(\frac{0.00238}{2}\right)V_{rel}^2(9)(50)(2\text{ rotors})$$

$$= \text{Drag} = C_d(\rho/2)V^2A_{wetted} = (0.005)(1.99/2)(3500)V^2,\quad \text{solve } \mathbf{V_{rel} \approx 1.23V}$$

$$\text{Law of cosines:}\quad W^2 = V^2 + V_{rel}^2 - 2VV_{rel}\cos(\theta + 90°),$$

$$\text{or:}\quad (20)^2 = V^2 + (1.23V)^2 - 2V(1.23V)\cos(111.8°),\quad \text{solve for } \mathbf{V_{ship} \approx 10.8\ \frac{ft}{s}}\quad \textit{Ans.}$$

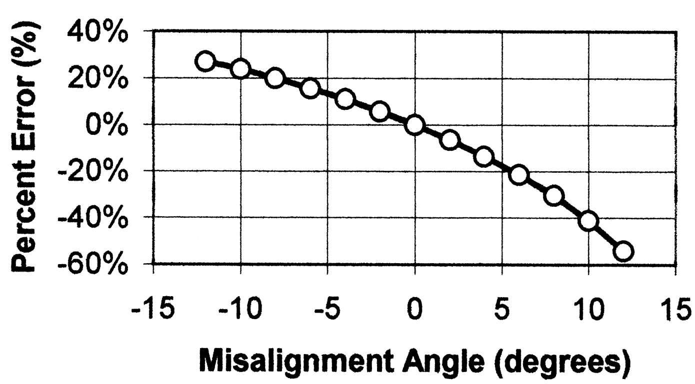
8.56 A proposed freestream velocimeter would use a cylinder with pressure taps at $\theta =$ 180° and at 150°. The pressure difference would be a measure of stream velocity U_∞. However, the cylinder must be aligned so that one tap exactly faces the freestream. Let the misalignment angle be δ, that is, the two taps are at (180° + δ) and (150° + δ). Make a plot of the percent error in velocity measurement in the range –20° < δ < +20° and comment on the idea.

Solution: Recall from Eq. (8.34) that the surface velocity on the cylinder equals $2U_\infty\sin\theta$. Apply Bernoulli's equation at both points, 180° and 150°, to solve for stream velocity:

$$p_1 + \frac{\rho}{2}[2U_\infty\sin(180° + \delta)]^2 = p_2 + \frac{\rho}{2}[2U_\infty\sin(150° + \delta)]^2,$$

$$\text{or:}\quad U_\infty = \frac{\sqrt{\Delta p/2\rho}}{\sqrt{\sin^2(150° + \delta) - \sin^2(180° + \delta)}}$$

The error is zero when $\delta = 0°$. Thus we can plot the percent error versus δ. When $\delta = 0°$, the denominator above equals 0.5. When $\delta = 5°$, the denominator equals 0.413, giving an error on the low side of (0.413/0.5) – 1 = –17%! The plot below shows that this is a very poor idea for a velocimeter, since even a small misalignment causes a large error.



Problem 8.56

8.57 In principle, it is possible to use rotating cylinders as aircraft wings. Consider a cylinder 30 cm in diameter, rotating at 2400 rev/min. It is to lift a 55-kN airplane flying at 100 m/s. What should the cylinder length be? How much power is required to maintain this speed? Neglect end effects on the rotating wing.

Solution: Assume sea-level air, $\rho = 1.23\ \text{kg/m}^3$. Use Fig. 8.11 for lift and drag:

$$\frac{a\omega}{U_\infty} = \frac{(0.15)[2400(2\pi/60)]}{100} \approx 0.38. \quad \text{Fig. 8.11: Read } C_L \approx 1.8,\ C_D \approx 1.1$$

$$\text{Then}\quad \text{Lift} = 55000\ \text{N} = C_L\frac{\rho}{2}U_\infty^2 DL = (1.8)\left(\frac{1.23}{2}\right)(100)^2(0.3)L,$$

solve **L ≈ 17 m** *Ans.*

$$\text{Drag} = C_D\frac{\rho}{2}U_\infty^2 DL = (1.1)\left(\frac{1.23}{2}\right)(100)^2(0.3)(17) \approx 33600\ \text{N}$$

Power required = FU = (33600)(100) ≈ **3.4 MW!** *Ans.*

The power requirements are ridiculously high. This airplane has way too much drag.

8.58 Plot the streamlines due to a line sink (–*m*) at the origin, plus line sources (+*m*) at (*a*, 0) and (4*a*, 0). *Hint*: A cylinder of radius 2*a* appears.

Solution: The overall stream function is

$$\psi = m\tan^{-1}\left(\frac{y-4a}{x}\right) + m\tan^{-1}\left(\frac{y-a}{x}\right) - m\tan^{-1}(y/x)$$

The cylinder shape, of radius 2*a*, is the streamline $\psi = -\pi/2$. *Ans.*

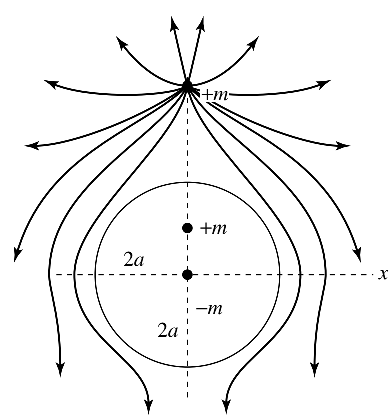


Fig. P8.58

8.59 By analogy with Prob. 8.58 above, plot the streamlines due to counterclockwise line vortices +K at (0, 0) and (4a, 0) plus a clockwise line vortex (–K) at (a, 0). *Hint*: Again a cylinder of radius 2*a* appears.

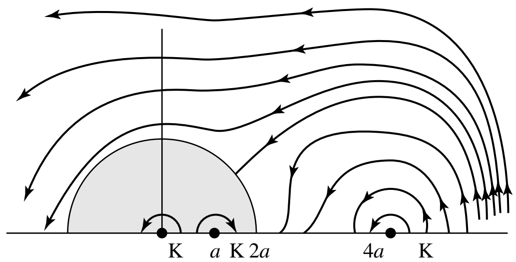


Fig. P8.59

8.60 One of the corner-flow patterns of Fig. 8.15 is given by the cartesian stream function $\psi = A(3yx^2 - y^3)$. Which one? Can this correspondence be proven from Eq. (8.49)?

Solution: This ψ is Fig. 8.15a, **flow in a 60° corner**. [Its velocity potential was given earlier Eq. (8.49) of the text.] The trigonometric form (Eq. 8.49 for n = **3**) is

$$\psi = Ar^3\sin(3\theta), \quad \text{but } \sin(3\theta) \equiv 3\sin\theta\cos^2\theta - \sin^3\theta.$$

Introducing $y = r\sin\theta$ and $x = r\cos\theta$, we obtain $\psi = \mathbf{A(3yx^2 - y^3)}$ *Ans.*

8.61 Plot the streamlines of Eq. (8.49) in the upper right quadrant for $n = 4$. How does the velocity increase with x outward along the x axis from the origin? For what corner angle and value of n would this increase be linear in x? For what corner angle and n would the increase be as x^5?

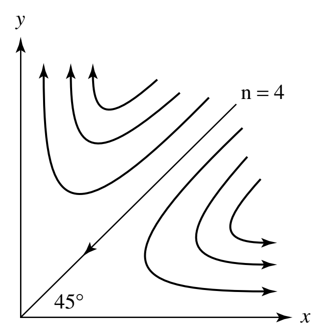


Fig. P8.61

Solution: For $n = 4$, we have **flow in a 45° corner**, as shown. Compute

$$n = 4: \quad \psi = Ar^4\sin(4\theta), \quad v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = 4Ar^3\cos(4\theta)$$

$$\text{Along the } x\text{-axis,} \quad \theta = 0,\ r = x,\ v_r = u = (\text{const})\boldsymbol{x^3} \quad \textit{Ans. (a)}$$

In general, for any n, the flow along the x-axis is $u = (\text{const})x^{n-1}$. Thus u is linear in x for $n = \mathbf{2}$ (a 90° corner). *Ans*. (b). And $u = Cx^5$ if $n = \mathbf{6}$ (a 30° corner). *Ans*. (c)

8.62 Combine stagnation flow, Fig. 8.14b, with a source at the origin:

$$f(z) = Az^2 + m\ln(z)$$

Plot the streamlines for $m = AL^2$, where L is a length scale. Interpret.

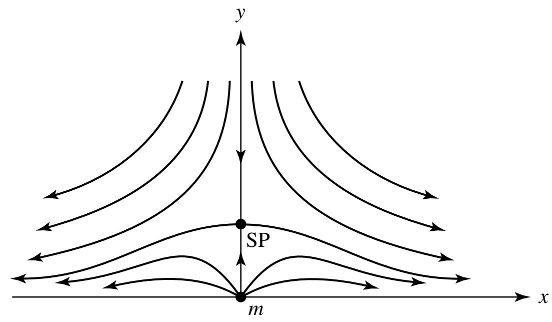


Fig. P8.62

Solution: The imaginary part of this complex potential is the stream function:

$$\psi = 2Axy + m\tan^{-1}\left(\frac{y}{x}\right), \quad \text{with } m = AL^2$$

The streamlines are shown on the previous page. The source pushes the oncoming stagnation flow away from the vicinity of the origin. There is a stagnation point above the source, at (x, y) = (**0, L/√2**). Thus we have "stagnation flow near a bump." *Ans.*

8.63 The superposition in Prob. 8.62 above leads to stagnation flow near a curved *bump*, in contrast to the flat wall of Fig. 8.15*b*. Determine the maximum height *H* of the bump as a function of the constants *A* and *m*. The bump crest is a stagnation point:

$$v_{\text{bump crest}} = -2AH + \frac{m}{H} = 0 \quad \text{whence } \boldsymbol{H_{\text{bump}} = \sqrt{\frac{m}{2A}}} \quad \textit{Ans.}$$

8.64 Consider the polar-coordinate velocity potential $\phi = Br^{1.2}\cos(1.2\theta)$, where B is a constant. (a) Determine whether $\nabla^2\phi = 0$. If so, (b) find the associated stream function $\psi(r, \theta)$ and (c) plot the full streamline which includes the x-axis ($\theta = 0$) and interpret.

Solution: (a) It is laborious, but the velocity potential satisfies Laplace's equation in polar coordinates:

$$\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial^2\phi}{\partial\theta^2}\right) \equiv 0 \quad \text{if } \boldsymbol{\phi = Br^{1.2}\cos(1.2\theta)} \quad \textit{Ans. (a)}$$

(b) This example is one of the family of "corner flow" solutions in Eq. (8.49). Thus:

$$\boldsymbol{\psi = Br^{1.2}\sin(1.2\theta)} \quad \textit{Ans. (b)}$$

(c) This function represents **flow around a 150° corner**, as shown below. *Ans.* (c)

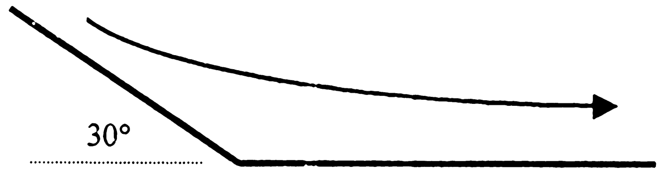


Fig. P8.64

8.65 Potential flow past a wedge of half-angle θ leads to an important application of laminar-boundary-layer theory called the *Falkner-Skan flows* [Ref. 15 of Chap. 8, pp. 242–247]. Let *x* denote distance along the wedge wall, as in Fig. P8.65, and let $\theta = 10°$. Use Eq. (8.49) to find the variation of surface velocity *U*(*x*) along the wall. Is the pressure gradient adverse or favorable?

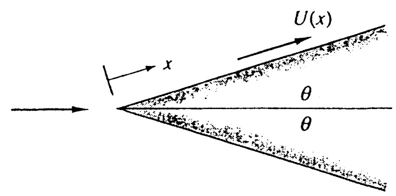


Fig. P8.65

Solution: As discussed above, all wedge flows are "corner flows" and have a velocity along the wall of the form $u = (\text{const})x^{n-1}$, where n = π/(turning angle). In this case, the turning angle is $\beta = (\pi - \theta)$, where $\theta = 10° = \pi/18$. Hence the proper value of n here is:

$$n = \frac{\pi}{\beta} = \frac{\pi}{\pi - \pi/18} = \frac{18}{17}, \quad \text{hence } U = Cx^{n-1} = \mathbf{Cx^{1/17}} \ \ \textbf{(favorable gradient)} \quad \textit{Ans.}$$

8.66 The inviscid velocity along the wedge in Prob. 8.65 has the form U(x) = Cx^m, where m = n − 1 and n is the exponent in Eq. (8.49). Show that, for any C and n, computation of the laminar boundary-layer by Thwaites' method, Eqs. (7.53) and (7.54), leads to a unique value of the Thwaites parameter λ. Thus wedge flows are called *similar* [Ref. 15 of Chap. 8, p. 244].

Solution: The momentum thickness is computed by Eq. (7.54), assuming $\theta_o = 0$:

$$\theta^2 = \frac{0.45\nu}{U^6}\int_0^x U^5\,dx = \frac{0.45\nu}{C^6x^{6m}}\int_0^x C^5x^{5m}\,dx = \frac{0.45\nu\, x^{1-m}}{C(5m+1)} \quad \text{Then use Eq. (7.53):}$$

$$\lambda = \frac{\theta^2}{\nu}\frac{dU}{dx} = \left(\frac{0.45x^{1-m}}{C(5m+1)}\right)(mCx^{m-1}) = \mathbf{\frac{0.45m}{5m+1}} \quad \text{(independent of C and m)} \quad \textit{Ans.}$$

8.67 Investigate the complex potential function $f(z) = U_\infty(z + a^2/z)$, where a is a constant, and interpret the flow pattern.

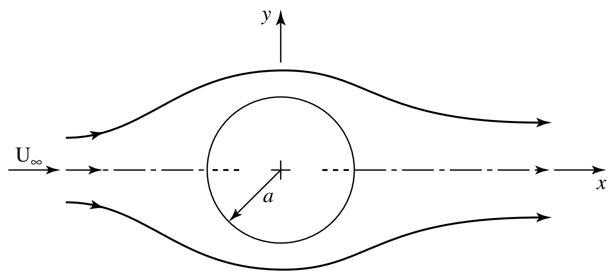


Fig. P8.67

Solution: This represents flow past a **circular cylinder** of radius a, with stream function and velocity potential identical to the expressions in Eqs. (8.31) and (8.32) with K = 0. [There is no circulation.]

8.68 Investigate the complex potential function $f(z) = U_\infty z + m\ \ln[(z + a)/(z - a)]$, where m and a are constants, and interpret the flow pattern.

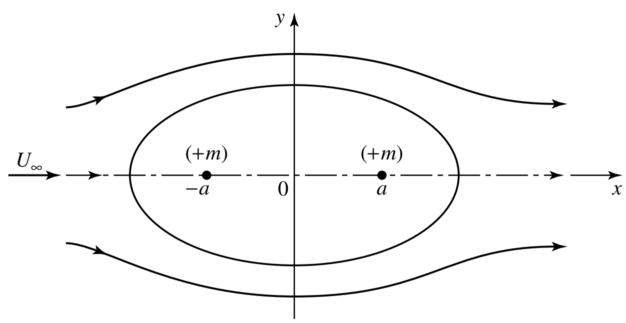


Fig. P8.68

Solution: This represents flow past a **Rankine oval**, with stream function identical to that given by Eq. (8.29).

8.69 Investigate the complex potential function $f(z) = A\cosh(\pi z/a)$, where a is a constant, and plot the streamlines inside the region shown in Fig. P8.69. What hyphenated French word might describe this flow pattern?

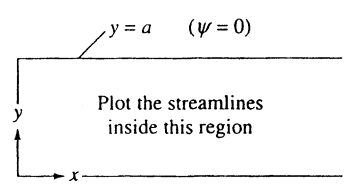
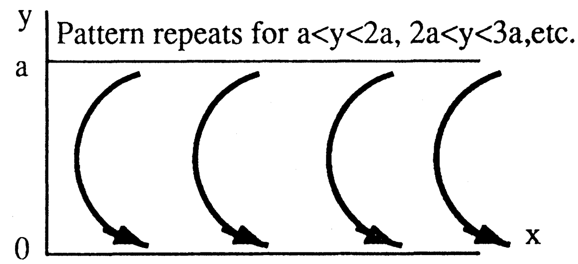


Fig. P8.69

Solution: This potential splits into

$$\psi = A\sinh(\pi x/a)\sin(\pi y/a)$$
$$\phi = A\cosh(\pi x/a)\cos(\pi y/a)$$

and represents flow in a "cul-de-sac" or blind alley.



8.70 Show that the complex potential $f(z) = U_\infty[z + (a/4)\coth(\pi z/a)]$ represents flow past an oval shape placed midway between two parallel walls $y = \pm a/2$. What is a practical application?

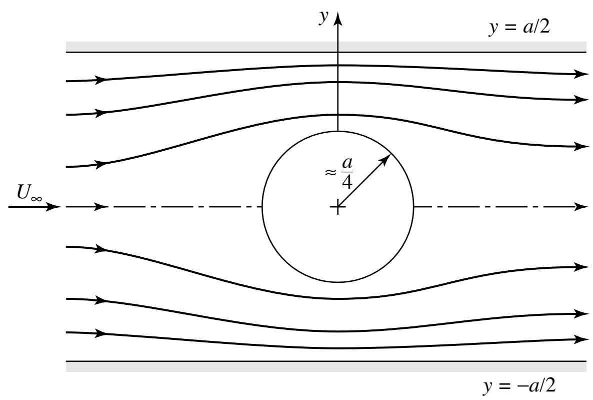


Fig. P8.70

Solution: The stream function of this flow is

$$\psi = U_\infty\left[y - \frac{(a/4)\sin(2\pi y/a)}{\cosh(2\pi x/a) - \cos(2\pi y/a)}\right]$$

The streamlines are shown in the figure. The body shape, trapped between y = ±a/2, is nearly a cylinder, with width a/2 and height 0.51a. A nice application is the estimate of wall "blockage" effects when a body (say, in a wind tunnel) is trapped between walls.

8.71 Figure P8.71 shows the streamlines and potential lines of flow over a thin-plate weir as computed by the complex potential method. Compare qualitatively with Fig. 10.16*a*. State the proper boundary conditions at all boundaries. The velocity potential has equally spaced values. Why do the flow-net "squares" become smaller in the overflow jet?

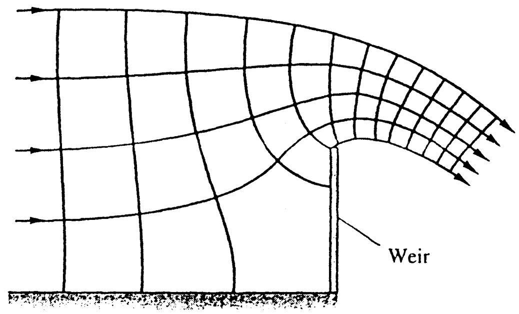
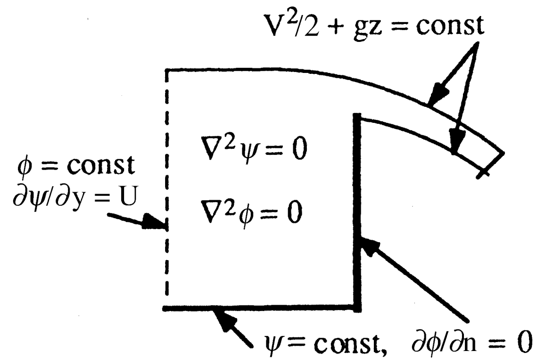


Fig. P8.71

Solution: Solve Laplace's equation for either ψ or ϕ (or both), find the velocities $u = \partial\phi/\partial x$, $v = \partial\phi/\partial y$, force the (constant) pressure to match Bernoulli's equation on the free surfaces (whose shape is *a priori* unknown). The squares become smaller in the overfall jet because the velocity is increasing.



8.72 Use the method of images to construct the flow pattern for a source *+m* near <u>two</u> walls, as in Fig. P8.72. Sketch the velocity distribution along the lower wall (y = 0). Is there any danger of flow separation along this wall?

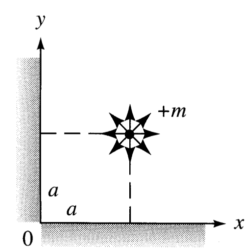
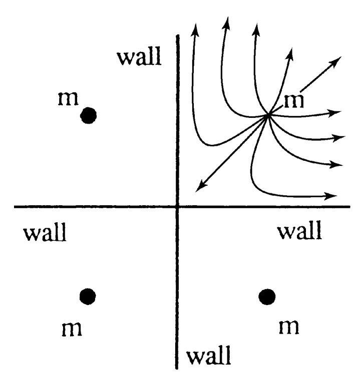


Fig. P8.72

Solution: This pattern is the same as that of Prob. 8.28. It is created by placing **four** identical sources at (x, y) = (±a, ±a), as shown. Along the wall ($x \geq 0$, $y = 0$), the velocity first increases from 0 to a maximum at x = a. Then the velocity *decreases* for x > a, which is an *adverse* pressure gradient—**separation may occur**. *Ans.*



8.73 Set up an image system to compute the flow of a source at *unequal* distances from *two* walls, as shown in Fig. P8.73. Find the point of maximum velocity on the y-axis.

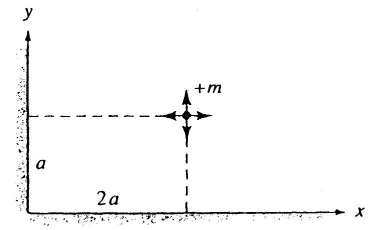
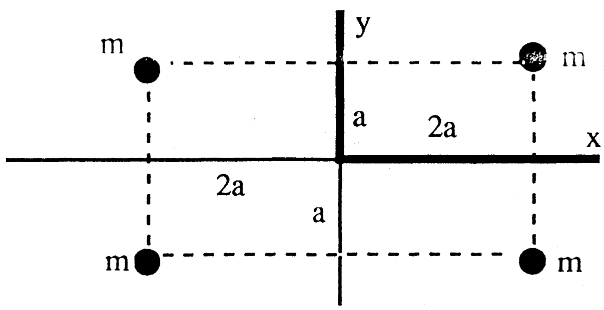


Fig. P8.73

Solution: Similar to Prob. 8.72 on the previous page, we place identical sources (+m) at the symmetric (but non-square) positions (x, y) = (±2a, ±a) as shown below. The induced velocity along the wall (x > 0, y = 0) has the form

$$U = \frac{2m(x+2a)}{(x+2a)^2 + a^2} + \frac{2m(x-2a)}{(x-2a)^2 + a^2}$$



This velocity has a maximum (to the *right*) at **x ≈ 2.93a**, **U ≈ 1.387 m/a**. *Ans.*

8.74 A positive line vortex *K* is trapped in a corner, as in Fig. P8.74. Compute the total induced velocity at point B, (x, y) = (2a, a), and compare with the induced velocity when no walls are present.

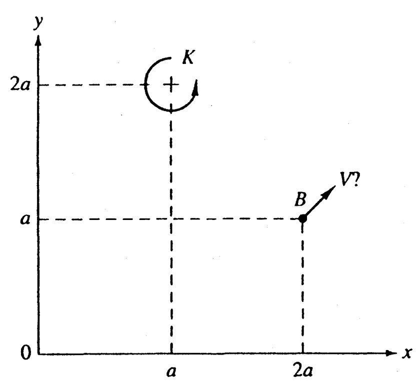
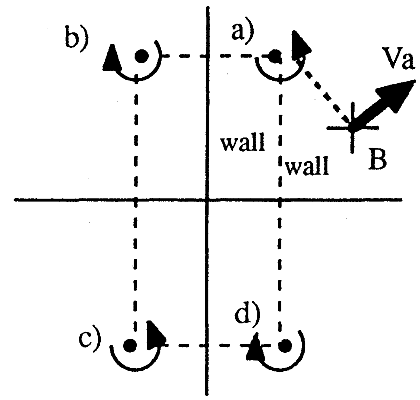


Fig. P8.74

Solution: The two walls are created by placing vortices, as shown at right, at (x, y) = (±a, ±2a). With only one vortex (#a), the induced velocity $\mathbf{V}_a$ would be



$$\mathbf{V}_a = \frac{K}{2a}\mathbf{i} + \frac{K}{2a}\mathbf{j}, \quad \text{or} \quad \frac{K}{a\sqrt{2}} \text{ at } 45^\circ \angle$$

as shown at right. With the walls, however, we have to add this vectorially to the velocities induced by vortices b, c, and d.

$$\text{With walls:} \quad \mathbf{V} = \sum \mathbf{V}_{a,b,c,d} = \frac{K}{a}\left(\frac{1}{2} - \frac{1}{10} - \frac{1}{6} + \frac{3}{10}\right)\mathbf{i} + \frac{K}{a}\left(\frac{1}{2} - \frac{3}{10} + \frac{1}{6} - \frac{1}{10}\right)\mathbf{j},$$

$$\text{or:} \quad \mathbf{V}_B = 0.533\frac{K}{a}\mathbf{i} + 0.267\frac{K}{a}\mathbf{j} = \frac{8K}{15a}\mathbf{i} + \frac{4K}{15a}\mathbf{j} \quad \textit{Ans.}$$

The presence of the walls thus causes a significant change in the magnitude and direction of the induced velocity at point B.

8.75 Using the four-source image pattern needed to construct the flow near a corner shown in Fig. P8.72, find the value of the source strength *m* which will induce a wall velocity of 4.0 m/s at the point $(x, y) = (a, 0)$ just below the source shown, if $a = 50$ cm.

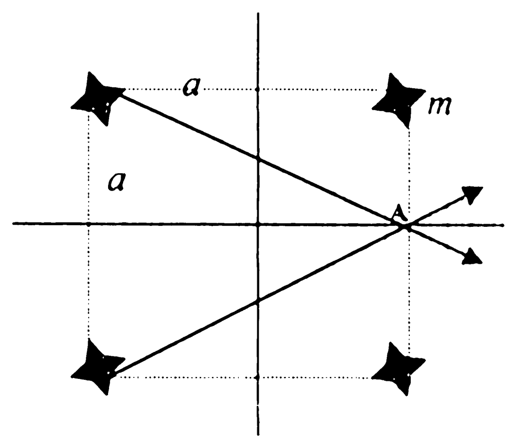


Fig. P8.75

Solution: The flow pattern is formed by four equal sources *m* in the 4 quadrants, as in the figure at right. The sources above and below the point A(*a*, 0) cancel each other at A, so the velocity at A is caused only by the two left sources. The velocity at A is the sum of the two horizontal components from these 2 sources:

$$V_A = 2\frac{m}{\sqrt{a^2+(2a)^2}}\frac{2a}{\sqrt{a^2+(2a)^2}} = \frac{4ma}{5a^2} = \frac{4m}{5(0.5m)} = 4\ \frac{m}{s} \quad \text{if } \boldsymbol{m = 2.5}\ \frac{\mathbf{m^2}}{\mathbf{s}} \qquad \textit{Ans.}$$

8.76 Use the method of images to approximate the flow past a cylinder at distance $4a$ from the wall, as in Fig. P8.76. To illustrate the effect of the wall, compute the velocities at points A, B, C, and D, comparing with a cylinder flow in an infinite expanse of fluid (without walls).

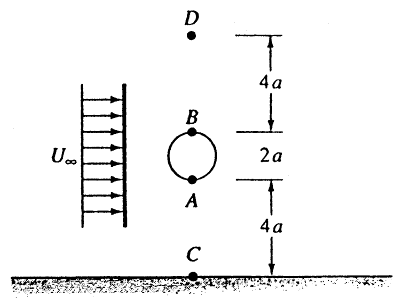


Fig. P8.76

Solution: Let doublet #1 be above the wall, as shown, and let image doublet #2 be below the wall, at (x, y) = (0, $-5a$). Then, at any point on the y-axis, the total velocity is

$$V_{x=0} = -v_\theta\big|_{90^\circ} = U_\infty[1 + (a/r_1)^2 + (a/r_2)^2]$$

Since the images are $10a$ apart, the cylinders are only slightly out-of-round and the velocities at A, B, C, D may be tabulated as follows:

Point:	**A**	**B**	**C**	**D**
r_1:	a	a	$5a$	$5a$
r_2:	$9a$	$11a$	$5a$	$15a$
V_{walls}:	$2.012U_\infty$	$2.008U_\infty$	$1.080U_\infty$	$1.044U_\infty$
$V_{no\ walls}$:	$2.0U_\infty$	$2.0U_\infty$	$1.04U_\infty$	$1.04U_\infty$

The presence of the walls causes only a slight change in the velocity pattern.

8.77 Discuss how the flow pattern of Prob. 8.58 might be interpreted to be an *image*-system construction for circular walls. Why are there two images instead of one?

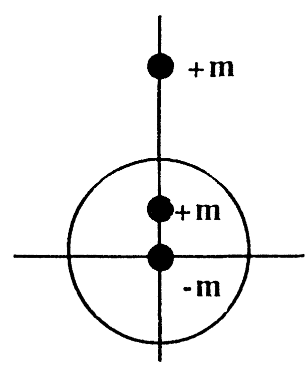


Fig. P8.77

Solution: The missing "image sink" in this problem is at y = +∞ so is not shown. If the source is placed at y = a and the image source at y = b, the radius of the cylinder will be $R = \sqrt{(\mathbf{ab})}$. For further details about this type of imaging, see Chap. 8, Ref. 3, p. 230.
